(ii) Decomposition by Multiplicative Hypothesis (Multiplicative Model).

The various components in a time series operate proportionately to the general level of the series, the traditional or classical multiplicative model is appropriate. According to the multiplicative model,

$$y_t = T_t \times S_t \times C_t \times R_t \quad \rightarrow (2)$$

where S_t, C_t and R_t, instead of assuming +ve or -ve values, are indices fluctuating above or below unity and the geometric means of S_t in a year, C_t in a cycle and R_t in a long term period are unity. In a time series with both +ve and -ve values, this model cannot be applied unless the time series is translated by adding a suitable +ve value. It may be pointed out that the multiplicative decomposition of a time series is same as the additive decomposition of logarithmic values of the original time series.

i.e., $\log y_t = \log T_t + \log S_t + \log C_t + \log R_t$.

Uses of Time Series:

1. It enables us to study the past behaviour of the phenomenon under consideration.
2. The segregation and study of the various components is of paramount importance to businessman in the planning of future operations and in the formulation of executive and policy decisions.

3. It helps to compare the actual current performance of accomplishments with the expected ones and analyse the cause of such variations, if any.

4. It enables us to predict or estimate or forcast the behaviour of the phenomenon in future which is very essential for business planning.

5. It helps us to compare the changes in the values of different phenomemon at different times or places etc.

Measurement of Trend

Trend can be studied by the following methods.

(i) Free hand curve fitting (Graphic Method)

A free hand smooth curve obtained on plotting the values y_t against 't' enables us to form an idea about the general trend of the series. Its draw-backs are

* This method is very subjective i.e., the bias of the persons handling the data plays a very important role and as such different trend curves will be obtained by different persons for the same set of data.

* It does not enable us to measure trend.

(ii) Method of Semi-Averages

In this method, the whole data is divided into two parts with respect to time, (e.g) if we are given y_t for t from 1991 - 2002, i.e., over a period of 12 years, the two equal parts will be the data from 1991 to 1996 and 1997 to 2002.

In case of odd number of years the two parts are obtained by omitting the values corresponding to the middle year. (e.g) for the data from 1991 to 2001, the value corresponding to middle year, i.e., 1996 being omitted. Next we compute arithmetic mean for each part and plot these two averages against the mid value of the respective time periods covered by each part. The line obtained on joining these two points is the required trend line and may be extended both ways to estimate future values.

Merits :

(i) As compared with graphic method, the obvious advantage of this method is its objectivity in the sense that everyone who applies it would get the same results. Moreover, we can also estimate the trend value.

(ii) It is readily comprehensible as compared to the method of least squares or moving average method.

Limitations :

This method assumes linear relationship between the plotted points which may not exist. Moreover, the limitation of arithmetic mean as an average also stand in its way.

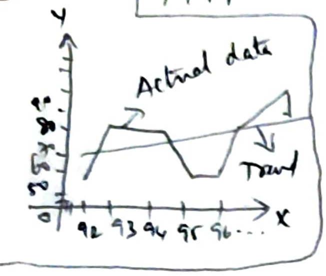
P6

1. Fit a trend line to the following data by the method of semi averages.

Year:	1992	93	94	95	96	97	98	99	2000	01	02	03	04
Bank clearences: (Rs in cr)	53	79	76	66	69	94	105	87	79	104	97	92	101

Sol:

Since $n = 13$, the two parts would consists 1992 to 1997 and 1999 to 2004, the year 1998 being omitted.



$\bar{x}_1$ = Avg. sales for 1^{st} part $= \frac{437}{6} = 72.83$ (Rs in cr)

$\bar{x}_2$ = Avg. sales for 2^{nd} part $= \frac{560}{6} = 93.33$ (Rs in cr).

$\bar{x}_1$ and $\bar{x}_2$ will be plotted against 1994 and 2001 respectively. Joining the points $A(1994, \bar{x}_1)$ and $B(2001, \bar{x}_2)$, we get the trend line.

(iii) Method of curve fitting by the principle of Least Squares:

The principle of LS is the most popular and widely used method of fitting mathematical functions to a given set of data. The method yields very correct results if sufficiently good appraisal of the form of the function to the fitted is obtained either by a scrutiny of the graphical plot of the values over time or by a theoretical understanding of the mechanism of the variable change. The various types of curves that may be used to describe the given data are (if y_t is the value of the variable corresponding to time t).

* A straight line ——— $y_t = a + bt$
* Second degree parabola ——— $y_t = a + bt + ct^2$
* k^{th} degree Polynomial ——— $y_t = a_0 + a_1 t + a_2 t^2 + \dots + a_k t^k$

* Exponential Curve ——— $y_t = ab^t \Rightarrow \log y_t = \log a + t \log b$
$= A + Bt$ (say)

* Second degree curve fitted to logarithms

$$y_t = a b^t c^{t^2}$$

$$\Rightarrow \log y_t = \log a + t \log b + t^2 \log c = A + Bt + Ct^2 \text{ (say)}$$

$\Rightarrow$ Fitting a st. line by LS method

Let the st. line trend be

$$y_t = a + bt \longrightarrow (1)$$

Principle of LS consists of minimizing the sum of squares of the deviations between the given values of y_t and their estimates given by (1).

In otherwords, we have to find a and b such that for given values of y_t corresponding to n different values of t,

$$E = \sum_t (y_t - a - bt)^2$$

is minimum. For maxima or minima of E, for variations in a & b, we have

$$\left.\begin{aligned} \frac{\partial E}{\partial a} = 0 &= -2\Sigma(y_t - a - bt) \Rightarrow \Sigma y_t = na + b\Sigma t \\ \frac{\partial E}{\partial b} = 0 &= -2\Sigma(t(y_t - a - bt)) \Rightarrow \Sigma t y_t = a\Sigma t + b\Sigma t^2 \end{aligned}\right\} \rightarrow (2)$$

Which are the normal equations for estimating a & b.

The values of Σy_t, Σt, Σt^2 are from the given data and (2) can be solved for a & b. With these values of a & b, line (1) gives the desired trend line.

Fitting of Second degree (parabolic) Trend :

Let the second degree trend curve be

$$y_t = a + bt + ct^2 \rightarrow (1)$$

Proceeding similarly as in the case of a straight line, the normal equations for estimating a, b and c are given by

$$\left.\begin{aligned} \Sigma y_t &= na + b\Sigma t + c\Sigma t^2 \\ \Sigma t y_t &= a\Sigma t + b\Sigma t^2 + c\Sigma t^3 \end{aligned}\right| \longrightarrow (2)$$

$$\Sigma t^2 y_t = a\Sigma t^2 + b\Sigma t^3 + c\Sigma t^4$$

the summation being taken over the values of the time series.

Fitting of Exponential Curve :

$$y_t = a b^t \rightarrow (3)$$

$$\Rightarrow \log y_t = \log a + t \log b$$

$$\Rightarrow Y = A + Bt \ (\text{say}) \rightarrow (4)$$

where $Y = \log y_t$; $A = \log a$ + $B = \log b \rightarrow (5)$

(5) is a st. line in t and Y and thus the normal equations for estimating A and B are

$$\left.\begin{aligned} \Sigma Y &= nA + B\Sigma t \\ \Sigma tY &= A\Sigma t + B\Sigma t^2 \end{aligned}\right\} \longrightarrow (6)$$

These equations can be solved for A and B and finally on using (6), we get $a = \text{antilog}(A)$;
$b = \text{antilog}(B)$;

Trend fitting by the Principle of least Squares:

Merits:

1. Because of its mathematical character, this method completely eliminates the elements of subjective judgement or personal bias on the part of the investigator.

2. Unlike the method of moving averages, this method enable us to compute the trend value for all the given time periods in the series.

3. The curve fitting by the principle of least squares is the only technique which enables us to obtain the rate of growth per annum, for yearly data, if linear trend is fitted.

Demerits:

1. The method is quite tedious and time consuming as compared with other methods.

2. The addition of even a single new observation necessitates all calculations to be done afresh.

3. Future prediction or forecasts based on this method are based only on the long term variation.

4. It cannot be used to fit growth curves like modified exponential curve, Logistic curve and Gompertz Curve.